A Survey Report of Initial Value Problem with Boundary Value Problems

Y. Madhavi Reddy, P. Venkata Ramana

Abstract: In mathematics, an initial value problem is an ordinary differential equation together with a specified value, called the initial condition, of the unknown function at a given point in the domain of the solution. In physics or other sciences, modeling a system frequently amounts to solving an initial value problem; in this context, the differential equation is an evolution equation specifying how, given initial conditions, the system will evolve with time. but In this paper specifies that various boundary conditions and boundary value problem with initial value problem.

Keywords- Boundary Condition

I. INTRODUCTION

An initial value problem is a differential equation

$y'(t) = f(t, y(t))$ with

$f : \Omega \subset \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^n$ where Ω is an open set

together with a point in the domain of *f*

$(t_0, y_0) \in \Omega,$

called the initial condition.

A solution to an initial value problem is a function *y* that is a solution to the differential equation and satisfies

$y(t_0) = y_0.$

This statement subsumes problems of higher order, by interpreting *y* as a vector. For derivatives of second or higher order, new variables (elements of the vector *y*) are introduced.

More generally, the unknown function *y* can take values on infinite dimensional spaces, such as Banach spaces or spaces of distributions.

1.1.Definition

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II. EXISTENCE AND UNIQUENESS OF SOLUTIONS

For a large class of initial value problems, the existence and uniqueness of a solution can be illustrated through the use of a calculator.

The Picard–Lindelöf theorem guarantees a unique solution on some interval containing t_0 if *f* is continuous on a region containing t_0 and y_0 and satifies the Lipschitz condition on the variable *y*. The proof of this theorem proceeds by reformulating the problem as an equivalent integral equation. The integral can be considered an operator which maps one function into another, such that the solution is a fixed point of the operator. The Banach fixed point theorem is then invoked to show that there exists a unique fixed point, which is the solution of the initial value problem.

An older proof of the Picard–Lindelöf theorem constructs a sequence of functions which converge to the solution of the integral equation, and thus, the solution of the initial value problem. Such a construction is sometimes called "Picard's method" or "the method of successive approximations". This version is essentially a special case of the Banach fixed point theorem.

Hiroshi Okamura obtained a necessary and sufficient condition for the solution of an initial value problem to be unique. This condition has to do with the existence of a Lyapunov function for the system.

In some situations, the function *f* is not of class C^1, or even Lipschitz, so the usual result guaranteeing the local existence of a unique solution does not apply. The Peano existence theorem however proves that even for *f* merely continuous, solutions are guaranteed to exist locally in time; the problem is that there is no guarantee of uniqueness. The result may be found in Coddington & Levinson (1955, Theorem 1.3) or Robinson (2001, Theorem 2.6). An even more general result is the Carathéodory existence theorem, which proves existence for some discontinuous functions *f*.

III. EXPONENTIAL SMOOTHING

Exponential smoothing is a general method for removing noise from a data series, or producing a short term forecast of time series data.

Single exponential smoothing is equivalent to computing an exponential moving average. The smoothing parameter is determined automatically, by minimizing the squared difference between the actual and the forecast values. Double exponential smoothing introduces a linear trend, and so has two parameters. For estimating initial value there are several methods. like we use these two formulas;

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$$y_0' = \left(\frac{\alpha}{1-\alpha}\right) a_t + b_t$$
$$y_0'' = \left(\frac{\alpha}{1-\alpha}\right) a_t + 2b_t$$

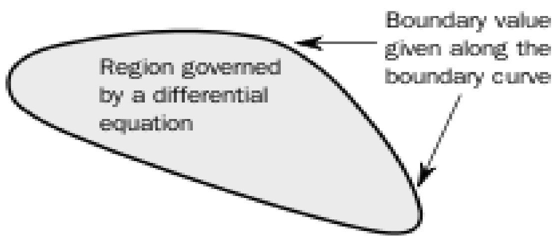
IV. BOUNDARY VALUE PROBLEM

In mathematics, in the field of differential equations, a boundary value problem is a differential equation together with a set of additional restraints, called the boundary conditions. A solution to a boundary value problem is a solution to the differential equation which also satisfies the boundary conditions.

Boundary value problems arise in several branches of physics as any physical differential equation will have them. Problems involving the wave equation, such as the determination of normal modes, are often stated as boundary value problems. A large class of important boundary value problems are the Sturm–Liouville problems. The analysis of these problems involves the eigenfunctions of a differential operator.

To be useful in applications, a boundary value problem should be well posed. This means that given the input to the problem there exists a unique solution, which depends continuously on the input. Much theoretical work in the field of partial differential equations is devoted to proving that boundary value problems arising from scientific and engineering applications are in fact well-posed.

Among the earliest boundary value problems to be studied is the Dirichlet problem, of finding the harmonic functions (solutions to Laplace's equation); the solution was given by the Dirichlet's principle.



Explanation

Boundary value problems are similar to initial value problems. A boundary value problem has conditions specified at the extremes ("boundaries") of the independent variable in the equation whereas an initial value problem has all of the conditions specified at the same value of the independent variable (and that value is at the lower boundary of the domain, thus the term "initial" value).

For example, if the independent variable is time over the domain [0,1], a boundary value problem would specify values for $y(t)$ at both $t = 0$ and $t = 1$, whereas an initial value problem would specify a value of $y(t)$ and $y'(t)$ at time $t = 0$.

Finding the temperature at all points of an iron bar with one end kept at absolute zero and the other end at the freezing point of water would be a boundary value problem.

If the problem is dependent on both space and time, one could specify the value of the problem at a given point for all time the data or at a given time for all space.

Concretely, an example of a boundary value (in one spatial dimension) is the problem

$$y''(x) + y(x) = 0$$

to be solved for the unknown function $y(x)$ with the boundary conditions

$$y(0) = 0, \ y(\pi/2) = 2.$$

Without the boundary conditions, the general solution to this equation is

$$y(x) = A\sin(x) + B\cos(x).$$

From the boundary condition $y(0) = 0$ one obtains

$$0 = A \cdot 0 + B \cdot 1$$

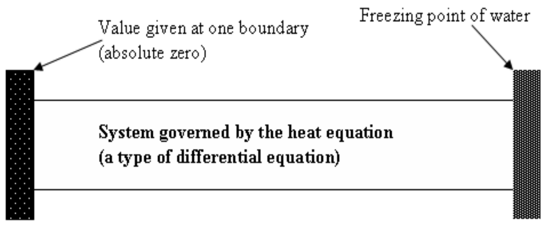
which implies that $B = 0$. From the boundary condition $y(\pi/2) = 2$ one finds

$$2 = A \cdot 1$$

and so $A = 2$. One sees that imposing boundary conditions allowed one to determine a unique solution, which in this case is

$$y(x) = 2\sin(x).$$

V. TYPES OF BOUNDARY VALUE PROBLEMS



The boundary value problem for an idealised 2D rod If the boundary gives a value to the normal derivative of the problem then it is a Neumann boundary condition. For example, if there is a heater at one end of an iron rod, then energy would be added at a constant rate but the actual temperature would not be known.

If the boundary gives a value to the problem then it is a Dirichlet boundary condition. For example, if one end of an iron rod is held at absolute zero, then the value of the problem would be known at that point in space.

If the boundary has the form of a curve or surface that gives a value to the normal derivative and the problem itself then it is a Cauchy boundary condition.

Aside from the boundary condition, boundary value problems are also classified according to the type of differential operator involved. For an elliptic operator, one discusses elliptic boundary value problems. For an hyperbolic operator, one discusses hyperbolic boundary value problems. These categories are further subdivided into linear and various nonlinear types.

5.1.Directional derivative

In mathematics, the directional derivative of a multivariate differentiable function along a given vector **v** at a given point **x** intuitively represents the instantaneous rate of change of the function, moving through **x** with a velocity specified by **v**. It therefore generalizes the notion of a partial derivative, in which the rate of change is taken along one of the coordinate curves, all other coordinates being constant.

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The directional derivative of a scalar function

$$f(\mathbf{x}) = f(x_1, x_2, \ldots, x_n)$$

along a vector

$$\mathbf{v} = (v_1, \ldots, v_n)$$

is the function defined by the limit[1]

$$\nabla_{\mathbf{v}} f(\mathbf{x}) = \lim_{h \to 0} \frac{f(\mathbf{x} + h\mathbf{v}) - f(\mathbf{x})}{h}.$$

If the function *f* is differentiable at **x**, then the directional derivative exists along any vector **v**, and one has

$$\nabla_{\mathbf{v}} f(\mathbf{x}) = \nabla f(\mathbf{x}) \cdot \mathbf{v}$$

where the ∇ on the right denotes the gradient and **.** is the dot product.[2] At any point **x**, the directional derivative of *f* intuitively represents the rate of change of *f* with respect to time when it is moving at a speed and direction given by **v** at the point **x**.

Some authors define the directional derivative to be with respect to the vector **v** after normalization, thus ignoring its magnitude. In this case, one has

$$\nabla_{\mathbf{v}} f(\mathbf{x}) = \lim_{h \to 0} \frac{f(\mathbf{x} + h\mathbf{v}) - f(\mathbf{x})}{h|\mathbf{v}|},$$

or in case *f* is differentiable at **x**,

$$\nabla_{\mathbf{v}} f(\mathbf{x}) = \nabla f(\mathbf{x}) \cdot \frac{\mathbf{v}}{|\mathbf{v}|}.$$

This definition has some disadvantages: it applies only when the norm of a vector is defined and nonzero. It is incompatible with notation used in some other areas of mathematics, physics and engineering, but should be used when what is wanted is the rate of increase in *f* per unit distance.

VI. CONCLUSION

Mainly in this paper we explain the initial value problem and boundary problem. mainly this paper theatrically focus on the initial value problem. This intial problem focuses on the various boundary conditions. But here we focus on the directional directive condition. This condition also uses in the differential equations problems.Now a days In mathematics we can use differential equations in more conditions.based upon the these paper we can also use boundary value problems.

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